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Characterization of Fracture Behavior in Repaired Skin/Stiffener Structure with an Inclined Central Crack

Ki-Hyun Chung*, **Won-Ho Yang**, **Sung-Pil Heo**
Department of Mechanical Engineering, Sungkyunkwan University, 300 Chunchun-Dong, Jangan-Ku, Suwon, Kyunggi-do 440-746, Korea

Finite element analysis for the stress intensity factor (SIF) at the skin/stiffener structure with inclined central crack repaired by composite stiffened panels is developed. A numerical investigation was conducted to characterize the fracture behavior and crack growth behavior at the inclined crack. In order to investigate the crack growth direction, maximum tangential stress (MTS) criterion are used. Also, this paper is to study the performance of the effective bonded composite patch repair of a plate containing an inclined central through-crack. The main objective of this research is the validation of the inclined crack patching design. In this paper, the reduction of stress intensity factors at the crack-tip and prediction of crack growth direction are determined to evaluate the effects of various non-dimensional design parameter including; composite patch thickness and stiffener distance. We report the results of finite element analysis on the stiffener locations and crack slant angles and discuss them in this paper. The research on cracked structure subjected to mixed mode loading is accomplished and concludes that more work using a different approaches is necessary. The authors hope the present study will aid those who are responsible for the repair of damaged aircraft structures and also provide general repair guidelines.

Key Words : Fracture Mechanics Analysis, Skin/Stiffener, Maximum Tangential Stress (MTS), Crack Growth Direction, Reduction of Stress Intensity Factor

1. Introduction

Due to the rapid development of aerospace industry, many investigators have studied the cracked structures by the request of safety. In the view of increasing the service life and reducing the repair cost, the proper repair methods have been suggested. As a simple and handy method, composite bonded patches, which now are widely used for cracked structures, can be used to repair or reinforce aerospace structures by modifying their load distribution and bypassing defects or cracks. In view of the rapidly increasing use of high strength, stiffness and low weight, fiber-reinforced composite material in advanced engineering structures such as high-performance aircraft is developed and used. Damage tolerance design and reliability of the composite structures have been of significant concern and have also brought a renewed interest in the theoretical analysis. Skin/stiffener structures are common features of airframes (e.g. fuselages) and wings are frequently made from stiffened sheets. Cracks can occur in such structures in the vicinity of the stiffener.

Baker and Jones (1988) expressed the many advantages of employing composite material patches for the bonded repair of cracked and damaged metallic structures. Bonded repairs are light-weight, eliminate unnecessary fastener holes in an already weakened and damaged structure,

* Corresponding Author,
E-mail : chungkh@nature. skku. ac. kr
TEL : +82-31-290-7496; **FAX** : +82-31-290-5849
Department of Mechanical Engineering, Sungkyunkwan University, 300 Chunchun-Dong, Jangan-Ku, Suwon, Kyunggi-do 440-746, Korea. (Manuscript **Received** November 30, 2000; **Revised** March 2, 2002)

enable load transfer more evenly and over large area, thus enhancing the fatigue life of the repaired structure. The primary advantage of composite repairs to cracked structures is to improve the damage tolerance of the repaired structure. To this aim, it is essential to demonstrate by fracture analysis and test that the repair can retain the crack propagation and damage tolerance requirements. But, this method is very difficult to exactly investigate the crack behavior. So, the experimental investigation and numerical method (Finite Element Method and Boundary Element Method etc.) are continuously accomplished to problems such as how to distribute stress or how to restrain the crack, and how to predict the crack propagation direction.

However, for a successful implementation of this repair technique, a thorough understanding of the effect of various design parameters of repair on the crack-tip stress intensity factors is necessary. Jone and Callinan (1979, 1983) studied the cracked patching using the finite element method. Chu and Ko (1989) proposed a method using collapsed isoparametric element to preserve the singular stress characteristic at the crack tip. But this method requires large number of nodal degrees of freedom. To overcome this problem, Atluri (1992) suggested the finite element alternating method (FEAM) in which the mesh needs not be very refined in the region of cracks. Chung and Yang (2000) studied the patch's efficiency in view of fracture mechanics and debonding. And they suggested the optimal patch shape on the reduction of stress intensity factor. A series of previously reported results have some limitation on the hypothesis that the structure is subjected uniaxial loading, but most of the structural components are subjected to biaxial loading. As the crack does not align with one of the principal directions, the mixed mode behavior will have a significant effect on the crack growth and fracture mechanics (Chue, et al., 1994).

In this paper, the analysis of repaired skin/stiffener structure with an inclined central crack is reported. The fracture mechanics analysis at the crack tip is performed and expectation of the crack propagation direction is to be suggested.

2. Method of Analysis

A schematic diagram of the skin/stiffener structure to be studied is shown in Fig. 1. An aluminum rectangular skin and I-type stiffener possess a centrally located horizontal through-thickness inclined crack. To prevent the propagation of crack, a nontapered $[0/90]_s$ boron/epoxy composite patch is considered. From the study by Chung et al. (2000) considering both fracture mechanics and debonding, tapered patch shape is more effective than nontapered patch. Skin/stiffener and skin/patch are bonded with epoxy. The thicknesses plate, adhesive layer and patch are 3mm, 0.1mm and 3mm. The other dimensions and the material properties are given in Fig. 2 and Table 1, respectively.

It is assumed that the skin/stiffener supports an uniform tensile stress (σ_0) of 10 MPa in the y-

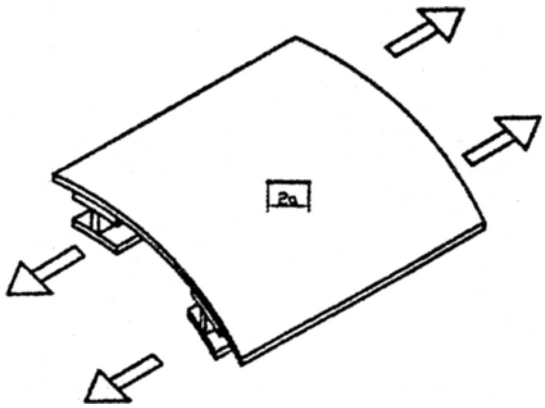


Fig. 1 Configuration of skin/stiffener plate

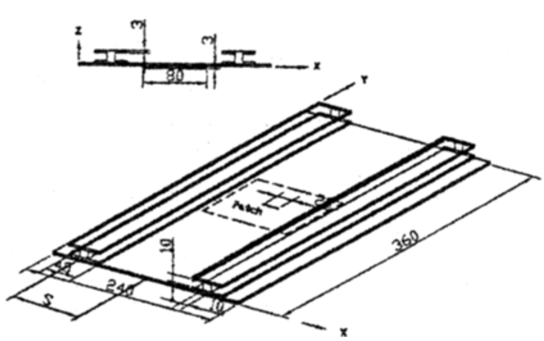


Fig. 2 Geometry of skin/stiffener plate with inclined central crack (unit : mm)

Table 1 Material properties of the aluminum, the boron/epoxy patch, and the adhesive layer

	Young's moduli (GPa)			Shear moduli (GPa)			Poisson's ratio		
	E_1	E_2	E_3	G_{12}	G_{13}	G_{23}	ν_{12}	ν_{13}	ν_{23}
Al-plate	71.02	—	—	—	—	—	0.32	—	—
Patch	208.1	8.18	8.18	7.24	7.24	4.94	0.1677	0.1677	0.035
Adhesive	2.2	—	—	—	—	—	0.32	—	—

direction. Owing to some of the limitations of analysis, this paper is based on the following simplified assumptions. (1) The curvature of the panel is neglected and is idealized as a flat panel. (2) The bonding of the patch and stiffener is perfect without debonding. The aluminum skin/stiffener, the boron/epoxy composite patch and epoxy layer must remain linear elastic. (3) The adhesive layer thickness is relatively thin compared to the plate/patch thickness, so that a generalized plane stress condition is considered. And the shear stress between plate and patch is treated as a body force.

The unpatch plate with an inclined central crack situation is considered first for validity of the finite element analysis. This analysis is made to evaluate the stress intensity factors and crack propagation direction of the plate under service loads in the absence of the patch and stiffener. The investigation of the patch efficiency, both of without patch and with patch are considered.

2.1 Stress intensity factor

The fracture parameter for the cracked structure is often given in terms of the stress intensity factor. The stress intensity factor for a central slant crack of length 2a in an infinite sheet subjected to a remote uniform uniaxial tensile stress is given by (Smith, 1988)

$$K_{\mathrm{I}}=\sigma\sqrt{\pi a}\sin^2\alpha \tag{1}$$

$$K_{\mathrm{II}}=\sigma\sqrt{\pi a}\sin\alpha\cos\alpha \tag{2}$$

where σ is the applied load, a is the half-crack length, and α is the angle between the crack line and the tensile axis.

The stress intensity factor can also be obtained by considering the displacement over the quarter-point crack-tip elements shown in Fig. 3(a) (Cook et al., 1989):

$$K_{\mathrm{I}}=\frac{\mu}{\kappa+1}\sqrt{\frac{2\pi}{l}}\left[(4v_{B2}-v_{C2})-(4v_{B1}-v_{C1})\right] \tag{3a}$$

$$K_{\mathrm{II}}=\frac{\mu}{\kappa+1}\sqrt{\frac{2\pi}{l}}\left[(4u_{B2}-u_{C2})-(4u_{B1}-u_{C1})\right] \tag{3b}$$

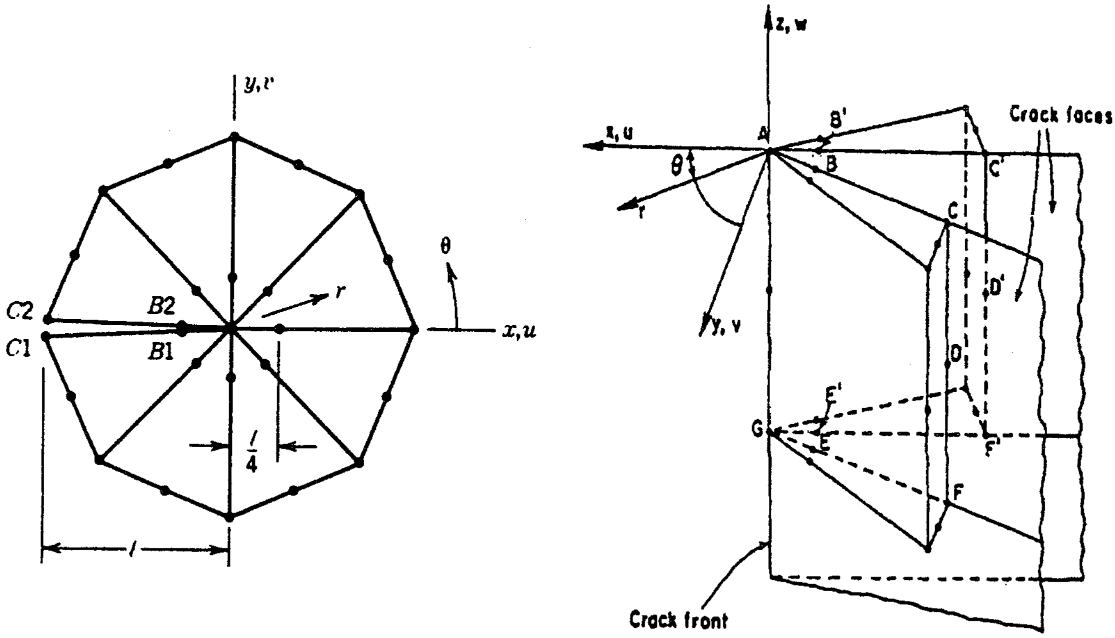
where, $\mu=E/2(1+\nu)$ is the shear modulus of elasticity, κ is equal to $(3-4\nu)$ for plane strain and $(3-\nu)/(1+\nu)$ for plane stress, and u_i, v_i are x-, y- components of the crack opening displacement (COD) at the collapsed crack tip elements.

Ingraffa and Manu (1980) proposed the calculation of stress intensity factor for the three-dimensional quarter-point crack-tip elements shown in Fig. 3(b).

$$\begin{aligned}K_{\mathrm{I}}=&\frac{E}{4(1-\nu^2)}\sqrt{\frac{\pi}{2L_1}}[(2v_B-v_C\\&+2v_E-v_F-2v_{B'}+v_{C'}-2v_{E'}\\&+v_{F'}-v_{D'})+\frac{1}{2}\eta(-4v_B+v_C\\&+4v_E-v_F+4v_{B'}-v_{C'}-4v_{E'}\\&+v_{F'})+\frac{1}{2}\eta^2(v_F+v_C-2v_D\\&-v_{F'}-v_{C'}+2v_{D'})]\end{aligned} \tag{4a}$$

$$\begin{aligned}K_{\mathrm{II}}=&\frac{E}{4(1-\nu^2)}\sqrt{\frac{\pi}{2L_1}}[(2u_B-u_C\\&+2u_E-u_F-2u_{B'}+u_{C'}-2u_{E'}\\&+u_{F'}-u_{D'})+\frac{1}{2}\eta(-4u_B+u_C\\&+4u_E-u_F+4u_{B'}-u_{C'}-4u_{E'}\\&+u_{F'})+\frac{1}{2}\eta^2(u_F+u_C-2u_D\\&-u_{F'}-u_{C'}+2u_{D'})]\end{aligned} \tag{4b}$$

where, E and ν are the Young's modulus and the Poisson's ratio, L_1 is the length of quarter-point element and η is the local coordinate at the crack front, respectively.



(a) (b)

Fig. 3 Arrangement of quarter-point wedge element along segment of crack front

2.2 Reduction of stress intensity factor

For the measure of the fracture mechanics safety and patching efficiency criteria at the repaired crack, the nondimensionalized reduction of stress intensity factor can be used such that

$$K^* = 1 - K_p/K_u \tag{5}$$

where, K_u, K_p are the stress intensity factors for the unpatched and patched crack plates.

The reduction of stress intensity factors is very important to design of repaired cracked plate because this value implies the patch efficiency.

As K^* increases the crack propagation decreases, on the other hand, as K^* decreases the possibility of fracture increases.

2.3 Prediction of crack growth direction

In many mixed-mode crack growth analysis, the prediction of crack growth direction is usually conducted only at the initial crack tip. A variety of theoretical models has been proposed for the prediction of fatigue crack growth direction under mixed-mode loadings.

The maximum tangential stress criterion (also called MTS) proposed by Erdogan and Sih (1963) is one of the earliest theories dealing with stable mixed-mode crack growth direction under static loading. It postulates that the crack will propagation in the direction governed by the maximum value of stress normal to the radial line from the crack tip. Therefore, this criterion assumes a mode I crack growth mechanism. Mathematically, condition for the crack growth direction can be expressed as:

$$\frac{\partial \sigma_\theta}{\partial \theta} = 0 \; ; \; \frac{\partial^2 \sigma_\theta}{\partial \theta^2} < 0 \tag{6a}$$

$$\sigma_c(\theta_c) = \sigma_c \tag{6b}$$

For mixed-mode, the crack growth angle θ based on this criterion is found from the following equation:

$$K_I\left(\sin\frac{\theta}{2} + \sin\frac{3\theta}{2}\right) + K_{II}\left(\cos\frac{\theta}{2} + \cos\frac{3\theta}{2}\right) = 0 \tag{7}$$

or

$$K_I \sin\theta + K_{II}(3\cos\theta - 1) = 0 \tag{8}$$

The minimum strain energy density criterion which is often called the S criterion was proposed by Sih (1974), and which is based on the local density of the energy field in the crack tip region.

The crack is assumed to grow in the direction along which the strain energy density reaches minimum value.

The strain energy density factor is determined by

$$S = a_{11}k_1^2 + 2a_{12}k_1k_2 + a_{22}k_2^2 + a_{33}k_3^2 \qquad (9)$$

where, a_{ij} can be expressed by angle(θ), Young's modulus(E) and Poisson's ratio(ν) and k_i are defined by

$$k_i = K_i/\sqrt{\pi}\,(i = \mathrm{I},\ \mathrm{II},\ \mathrm{III}) \qquad (10)$$

The condition for crack growth direction can be expressed as:

$$\frac{\partial S}{\partial \theta} = 0\ ;\ \frac{\partial^2 S}{\partial \theta^2} > 0 \qquad (11)$$

For mixed-mode loading, the following equation based on this criterion results:

$$\begin{aligned} &\sin 2\theta - 0.92 \sin \theta + 4R_k(\cos 2\theta - \cos \theta) \\ &+ R_k^2(0.92 \sin \theta - 3 \sin \theta) = 0 \end{aligned} \qquad (12)$$

where, R_k is the ratio $K_{\mathrm{II}}/K_{\mathrm{I}}$.

3. Finite Element Analysis

The basic geometry of cracked skin/stiffener structure considered in this study is shown in

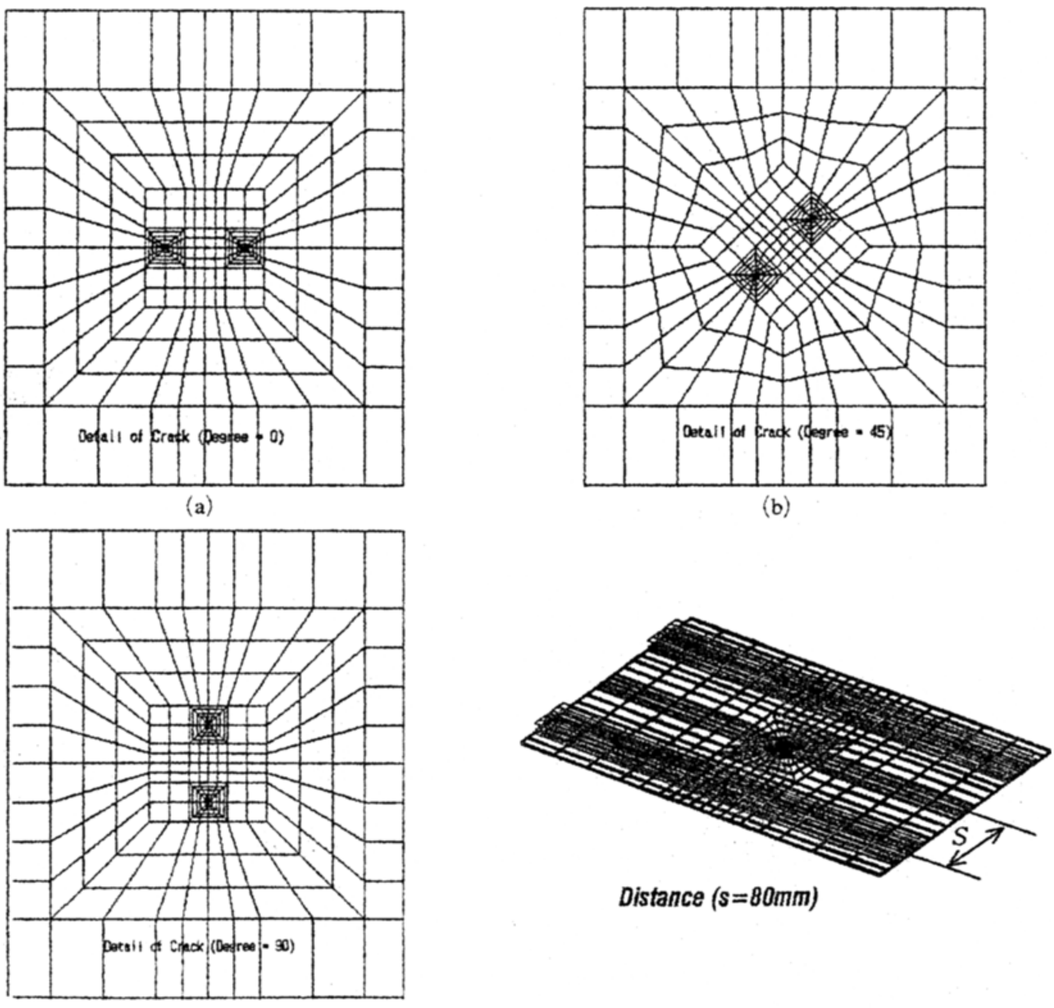


Fig. 4 Finite element modeling around crack, inclined degree(θ) (a) : 0°, (b) : 45°, (c) : 90°, (d) : whole modeling of S/a=8

Fig. 1. Consider a thin elastic aluminum sheet 240×360×3mm with an central crack of length *a* and I-type aluminum stiffener. The basic repair configuration is a 40×80×3mm boron/epoxy composite patch bonded by 0.3mm thick film-epoxy adhesive. Once an efficient model is established, we would investigate a set of this basic configuration to study the effect of stiffener distance and crack slant angle.

The slant cracked sheet is subjected to a remote uniaxial tensile load of 10 MPa. Since the problem has no plane of symmetry, it is necessary to model the whole structure by using three-dimensional 20-node isoparametric brick elements. The region adjacent to the crack front is modelled with the singular crack elements. In the singular element the mid-side nodes are shifted to the quarter-point position to induce the required $(1/r)^{1/2}$ stress singularity (Fig. 4). To get better results, the singular element sizes are kept within 10% of the crack length. Finite element analysis is done using a commercial ABAQUS code (version 5.8-8).

Figure 4 shows the finite element modeling for the stiffener distance 80mm and the detail configuration of the crack part with respect to crack angle.

4. Results and Discussion

4.1 Repair of inclined crack in unstiffened panels

Figures 5~6 show the stress intensity factors with respect to the inclined crack angle. The stress intensity factors are obtained in the average sense through the thickness. To investigate the validity of the results obtained, we compare those with other available results. As can be seen in Table 2 and Figs. 5~6, the results are in good agreement within 7~10% with those obtained by Smith (1988). These errors may be caused by the fact that the present study considers the finite model, but Smith's study considered the infinite model. Figures 7~8 show the nondimensional reduction of SIF with respect to various inclined crack angles. From the result, the patch is very efficient to restrain the crack growth. As compared with unpatched plate, the mode Ⅰ SIFs

Table 2 Comparison of stress intensity factor for inclined cracked rectangular plates (σ_0=10MPa, unit : MPa$\sqrt{mm}$)

Inclined Crack Angle(°)	Smith		Present	
	Mode Ⅰ SIFs	Mode Ⅱ SIFs	Mode Ⅰ SIFs	Mode Ⅱ SIFs
0	56.06	0	60.38	0
10	54.36	9.60	58.60	10.91
20	49.50	18.02	53.34	20.46
30	42.04	24.27	45.32	27.59
40	32.90	27.60	35.50	31.41
45	28.03	28.03	30.23	31.88
50	23.16	27.60	25.02	31.43
60	14.01	24.27	15.13	27.63
70	6.56	18.02	7.08	20.52
80	1.69	9.60	1.82	10.91
90	0	0	0	0

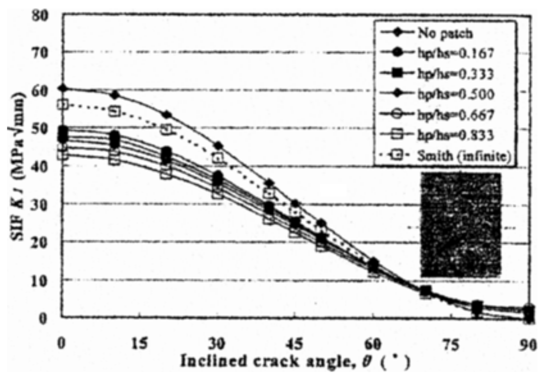


Fig. 5 Mode Ⅰ SIF with respect to inclined crack angle (σ_0=10MPa)

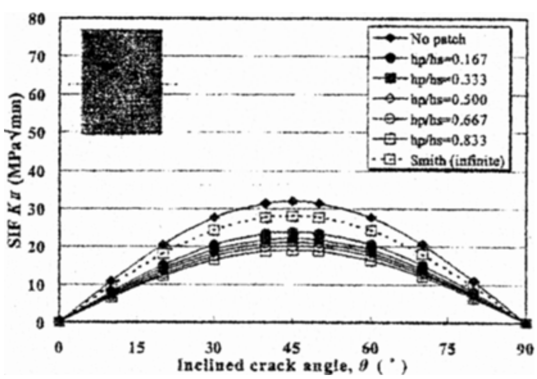


Fig. 6 Mode Ⅱ SIF with respect to inclined crack angle (σ_0=10MPa)

of patched plate are reduced about 20~30%. Mode Ⅰ SIFs of thick patch are rapidly decreased as the angle between loading direction and inclined crack is decreased. Oppositely, when the cracked inclined angle is over 75°, mode Ⅰ SIFs are increased as the thickness of patch increases. Especially, mode Ⅰ SIF is not zero at 90° of patched plate. This phenomenon is due to the presence of out-of-plane bending deformation which causes the nonlinear behavior of material and geometry.

Mode Ⅱ SIFs of patched plate are reduced a by about 30~45%. The reduction of mode Ⅱ SIFs is about 0.3~0.4, and whih is independent of inclined angle.

As can be concluded, the effect of thicker patch becomes even stronger, but the SIF does not grow infinitely as the patch thickness increases because of the out-of-plane bending deformation. And the effect of patch to the mode Ⅱ behavior is more significant than the mode Ⅰ behavior.

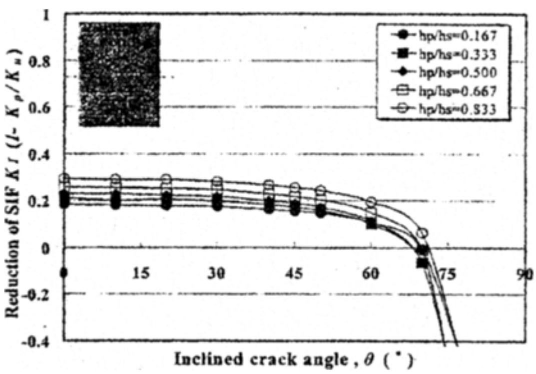


Fig. 7 Reduction of mode Ⅰ SIF with respect to inclined crack angle

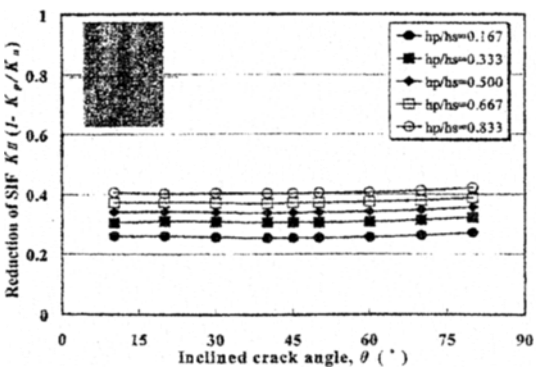


Fig. 8 Reduction of mode Ⅱ SIF with respect to inclined crack angle

4.2 Prediction of crack growth direction in unstiffened panels

Most of the studies of crack growth under mixed mode Ⅰ and Ⅱ loadings have been conducted using a plate with an inclined central crack under tension. To predict of crack growth direction, the maximum tangential stress criterion is examined.

Figure 9 shows the crack growth direction at the unrepaired and repaired plates with an inclined central crack. As can be seen, the patch effect to the crack growth direction is relatively slight. Together with the patch thickness increase, the crack growth direction has a tendency to grow toward the mode Ⅰ direction.

As inclined crack angle(θ) approaches about 50~60°, which the range is equal mode Ⅰ SIF with mode Ⅱ SIF, the crack growth direction of the plate repaired by composite patch becomes perpendicular to the apply loading direction.

When the inclined angle approaches the loading direction, the difference between the predicted crack growth direction of theoretical analysis and predicted direction of present analysis becomes langer. So, the crack growth direction is unstable.

4.3 Repair of inclined crack in stiffened panels

The influence of the distance of the stiffener

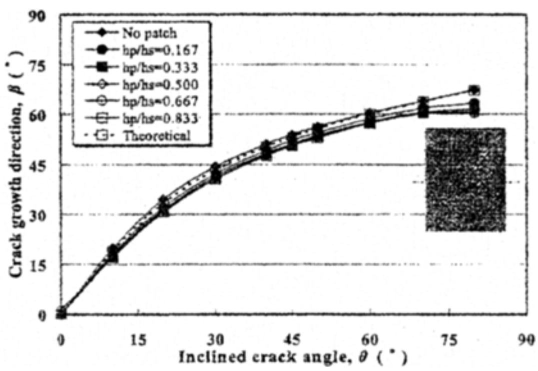


Fig. 9 Prediction of crack growth direction for the repaired plate

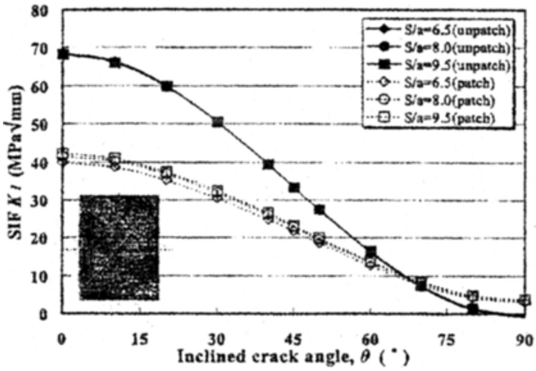


Fig. 10 Mode I SIF with respect to inclined crack angle (σ_0=10MPa)

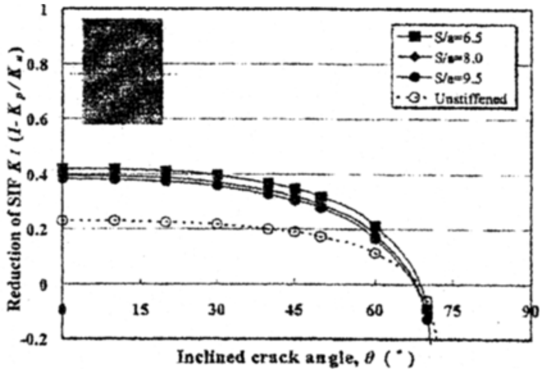


Fig. 12 Reduction of mode I SIF with respect to inclined crack angle

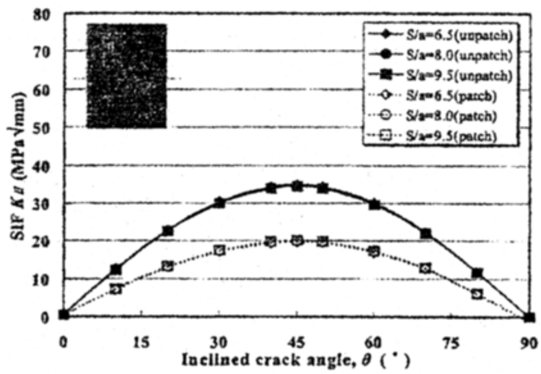


Fig. 11 Mode II SIF with respect to inclined crack angle (σ_0=10MPa)

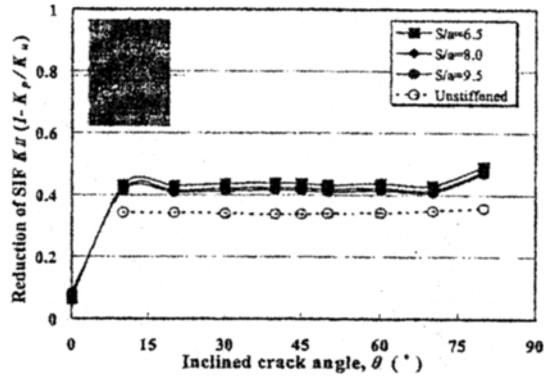


Fig. 13 Reduction of mode II SIF with respect to inclined crack angle

(S=65mm, 80mm and 95mm) on the predicted SIF and reduction of SIF is illustrated in Figs. 10~13. As can be seen, the stiffener location has less influence on the SIF. But, when the crack size increases, the distance of stiffener location might be playing the more important role in the crack restraint and crack growth direction. The SIF of patched skin/stiffener structure is decreased about 1.5 to 2 times as much as unpatched skin/stiffener structures. When inclined crack angle reached 90°, the stress intensity factor of mode I does not vanish.

In those figures, the stress intensity factors obtained are the maximum values through the plate thickness because the value distribution is very variable and the maximum value will cause the fracture. The stress intensity factor is not uniform along the thickness. It was found that the stress intensity factor of unpatched side is about 3 times as much as on the patched side.

The nondimensional maximum reduction of mode II SIF is about 0.45. These results emphasize the importance of the appropriate patch for preventing crack growth. But, the effect of panel to restrain the crack growth is more slight than the effect of patch.

4.4 Prediction of crack growth direction in stiffened panels

From the same method (MTS criterion) described above, we studied the prediction of crack growth direction in skin/stiffener structures with inclined central crack.

Figure 14 shows that the significant discrepancies occur when the inclined crack line is close to the tensile loading axis. Under mixed mode loadings, a crack does not necessarily propagate in mode I in the skin/stiffener structures. It is

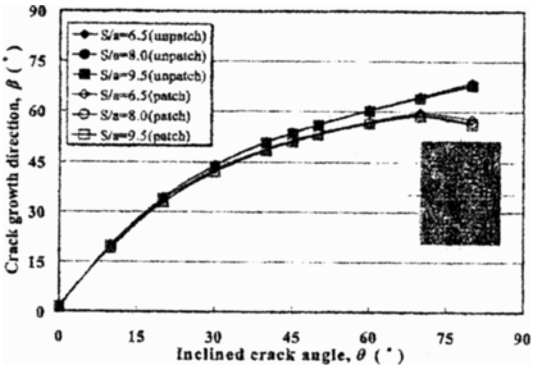


Fig. 14 Prediction of crack growth direction for repaired skin/stiffener

considered that the effect of stiffener appeares sensitively.

5. Conclusions

The objective of the study is the crack repair method and fracture mechanics analysis of skin/stiffener with inclined central crack to estimate the repair availability and to predict the crack growth direction. In this paper, we have only studied a few factor, so more extensive studies are needed to increase our understanding of the investigation of the crack behavior at the inclined central crack repaired by composite patch. The following conclusions are drawn from this study.

(1) The composite patch might be playing more important role in the crack restraint. The patch will reduce the stress intensity factor (mode Ⅰ : about 20~30%, mode Ⅱ : about 30~40%). Composite patch is effective to prevent the propagation of crack. As the thickness of the patch increases greater reduction of SIF is achieved by the crack patching.

(2) When the inclined crack angle approaches to 90°, mode Ⅰ stress intensity factors are not zero at the patched plate. This phenomenon is due to the presence of out-of-plane bending deformation which causes the nonlinear behavior of material and geometry.

(3) Inclined crack propagates to the perpendicular direction to the applied loading direction when mode Ⅰ SIF is equal to mode Ⅱ SIF. But, in the other case, crack does not necessarily propagate in mode Ⅰ in the skin/stiffener structures. This facts indicate that the effect of stiffener appears sensitively. The inclined crack has a tendency to growth toward the mode Ⅰ direction in with/without patches and stiffened/unstiffened structures.

(4) The influence of stiffener distance (S) is relatively small compared to the other factors. These results are caused by the fact that the stiffener distance is so far from the crack tip which calculated the stress intensity factor. But, it is considered that the effect of stiffener distance appears sensitively when the crack is more propagated.

(5) The panel plays an important role for preventing crack growth. But, the effect of the stiffener to restrain the crack growth is less than the effect of the patch.

(6) The following points are left as future studies: i) How to calculate the stress intensity factor at the thick patch? The stress intensity factor does not uniformly through-the-thickness. ii) More exact method to predict of the crack growth direction to the inclined crack plate must be developed.

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